NAG Toolbox for MATLAB

f08yf

1 Purpose

f08yf reorders the generalized Schur factorization of a matrix pair in real generalized Schur form.

2 Syntax

```
[a, b, q, z, ifst, ilst, info] = f08yf(wantq, wantz, a, b, q, z, ifst,
ilst, 'n', n)
```

3 Description

f08yf reorders the generalized real n by n matrix pair (S, T) in real generalized Schur form, so that the diagonal element or block of (S, T) with row index i_1 is moved to row i_2, using an orthogonal equivalence transformation. That is, S and T are factorized as

$$S = \hat{Q}\hat{S}\hat{Z}^{\mathrm{T}}, \qquad T = \hat{Q}\hat{T}\hat{Z}^{\mathrm{T}},$$

where $\left(\hat{S}, \hat{T}\right)$ are also in real generalized Schur form.

The pair (S, T) are in real generalized Schur form if S is block upper triangular with 1 by 1 and 2 by 2 diagonal blocks and T is upper triangular as returned, for example, by f08xa, or f08xe with **job** $=$ 'S'.

If S and T are the result of a generalized Schur factorization of a matrix pair (A, B)

$$A = QSZ^{\mathrm{T}}, \qquad B = QTZ^{\mathrm{T}}$$

then, optionally, the matrices Q and Z can be updated as $Q\hat{Q}$ and $Z\hat{Z}$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

5 Parameters

5.1 Compulsory Input Parameters

1: **wantq – logical scalar**

If **wantq** $=$ **true**, update the left transformation matrix Q.

If **wantq** $=$ **false**, do not update Q.

2: **wantz – logical scalar**

If **wantz** $=$ **true**, update the right transformation matrix Z.

If **wantz** $=$ **false**, do not update Z.

3: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The matrix S in the pair (S, T).

4: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1,\mathbf{n})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

The matrix T, in the pair (S,T).

5: **q(ldq,$*$) – double array**

The first dimension, **ldq**, of the array **q** must satisfy

if **wantq** $=$ **true**, **ldq** $\geq \max(1,\mathbf{n})$;
ldq ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1,\mathbf{n})$

If **wantq** $=$ **true**, the orthogonal matrix Q.

6: **z(ldz,$*$) – double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **wantz** $=$ **true**, **ldz** $\geq \max(1,\mathbf{n})$;
ldz ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1,\mathbf{n})$

If **wantz** $=$ **true**, the orthogonal matrix Z.

7: **ifst – int32 scalar**
8: **ilst – int32 scalar**

The indices i_1 and i_2 that specify the reordering of the diagonal blocks of (S,T). The block with row index **ifst** is moved to row **ilst**, by a sequence of swapping between adjacent blocks.

Constraint: $1 \leq \mathbf{ifst} \leq \mathbf{n}, 1 \leq \mathbf{ilst} \leq \mathbf{n}$.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The first dimension of the arrays **a**, **b** and the second dimension of the arrays **a**, **b**. (An error is raised if these dimensions are not equal.)

n, the order of the matrices S and T.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldq, ldz, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

The updated matrix $\hat{S}$.

2: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1,\mathbf{n})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

The updated matrix $\hat{T}$

3: **q(ldq,$*$) – double array**

The first dimension, **ldq**, of the array **q** must satisfy

if **wantq** $=$ **true**, **ldq** $\geq \max(1, \mathbf{n})$;
ldq ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **wantq** $=$ **true**, the updated matrix $Q\hat{Q}$.

If **wantq** $=$ **false**, **q** is not referenced.

4: **z(ldz,$*$) – double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **wantz** $=$ **true**, **ldz** $\geq \max(1, \mathbf{n})$;
ldz ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **wantz** $=$ **true**, the updated matrix $Z\hat{Z}$.

If **wantz** $=$ **false**, **z** is not referenced.

5: **ifst – int32 scalar**
6: **ilst – int32 scalar**

if **ifst** pointed on entry to the second row of a 2 by 2 block, it is changed to point to the first row; **ilst** always points to the first row of the block in its final position (which may differ from its input value by $+1$ or -1).

7: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **wantq**, 2: **wantz**, 3: **n**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **q**, 9: **ldq**, 10: **z**, 11: **ldz**, 12: **ifst**, 13: **ilst**, 14: **work**, 15: **lwork**, 16: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$

The transformed matrix pair $\left(\hat{S}, \hat{T}\right)$ would be too far from generalized Schur form; the problem is ill-conditioned. (S, T) may have been partially reordered, and **ilst** points to the first row of the current position of the block being moved.

7 Accuracy

The computed generalized Schur form is nearly the exact generalized Schur form for nearby matrices $(S + E)$ and $(T + F)$, where

$$\|E\|_2 = O\,\epsilon\|S\|_2 \qquad \text{and} \qquad \|F\|_2 = O\,\epsilon\|T\|_2,$$

and ϵ is the ***machine precision***. See Section 4.11 of Anderson *et al.* 1999 for further details of error bounds for the generalized nonsymmetric eigenproblem.

8 Further Comments

The complex analogue of this function is f08yt.

9 Example

```
wantq = false;
wantz = false;
a = [4, 1, 1, 2;
     0, 3, 4, 1;
     0, 1, 3, 1;
     0, 0, 0, 6];
b = [2, 1, 1, 3;
     0, 1, 2, 1;
     0, 0, 1, 1;
     0, 0, 0, 2];
q = [0, 0, 0, 0];
z = [0, 0, 0, 0];
ifst = int32(2);
ilst = int32(1);
[aOut, bOut, qOut, zOut, ifstOut, ilstOut, info] = ...
    f08yf(wantq, wantz, a, b, q, z, ifst, ilst)
```

```
aOut =
    4.1926    1.2591    2.5578    0.4520
    0.8712   -0.8627   -2.7912   -1.1383
         0         0    4.2426    2.1213
         0         0         0    6.0000
bOut =
    1.7439         0    0.7533    0.0661
         0   -0.5406   -1.8972   -1.7308
         0         0    2.1213    2.8284
         0         0         0    2.0000
qOut =
     0     0     0     0
zOut =
     0     0     0     0
ifstOut =
           2
ilstOut =
           1
info =
           0
```